

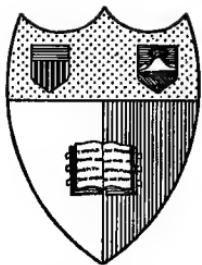
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SHADES AND SHADOWS AND PERSPECTIVE

A TEXT-BOOK
BASED ON THE PRINCIPLES OF
DESCRIPTIVE GEOMETRY

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P R E F A C E

THE aim of this treatise is to present, through a knowledge of descriptive geometry, those principles which are fundamental in the solution of both theoretical and practical problems and, by a formulation of these principles, to arrive at and thus place upon a mathematical basis those rules and formulas which are commonly used in practice.

The method of presentation, which has been developed and tested during fifteen years of experience in the class room, seeks to introduce and substantiate each principle by reference to the simplest form of magnitude. For this reason no attempt has been made at either elaborate or artistic drawings, as these belong rather to the field of application.

In the establishment of principles great effort is made to be explicit; but in the application of these principles the work is assigned in such a way as to call out the student's originality and demand of him work similar to that required in practice.

A feature of the book is a method of locating given parts, by which the work of the drawing and recitation room may be easily and definitely assigned. This reduces the labor of the instructor, makes it possible to assign special lay-outs for problems, and gives the student excellent practice in accuracy of measurement.

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SHADES AND SHADOWS, AND PERSPECTIVE

PART I

SHADES AND SHADOWS

CHAPTER I

DEFINITIONS AND ASSUMPTIONS

1. **The Subject defined.** — Shades and shadows is that branch of drawing which seeks, through a proper distribution of “light” and “shade,” to make a “graphic” representation more intelligible.

As the distribution of light and shade upon surfaces in nature is dependent upon the relation of these surfaces to some existing source of light, the problems of shades and shadows have to do with methods employed in the determination and the representation of (a) those portions of the surfaces of solids excluded from the light by the solids themselves and (b) those portions of other surfaces excluded from the light by the same solids.

2. **Ray.** — Light is supposed to emanate from luminous bodies in straight lines and in all directions. These lines are called *rays*.

3. **Umbra.** — That portion of space excluded from the light by an opaque body is called the *umbra* of the body.

The umbra of a point is a line ; the umbra of a line is in general a surface ; the umbra of a surface is in general a solid.

4. **Shade.** — That portion of the surface of a body excluded from the light by the body itself is called the *shade* of the body.

5. **Shadow.** — That portion of any surface excluded from the light by an external body is called the *shadow* of the body upon

that surface. The shadow of a body upon a surface is the intersection of the umbra of the body with that surface.

The shadow of a point is a point; the shadow of a line is in general a line; the shadow of a surface is in general a surface.

When the umbra intersects several surfaces, that intersection nearest the source of light is called the *primary shadow*, the next the *secondary shadow*, etc.

6. **Shade Line.** — The shade line is the boundary line of the shade; or, it is the line of separation between that portion of the surface which is in light and that portion which is in shade.

The shade line of a body is the locus of the points at which tangent rays are tangent to the body.

7. **Shadow Line.** — The shadow line is the boundary line of the shadow and is the shadow of the shade line; for the tangential rays pass through the points of the shade line, form the external rays of the umbra, and pierce the surface on which the shadow is cast in the outlines of the shadow.

8. **Plane of Rays.** — A plane of rays is any plane containing a ray.

9. **Pencil of Rays.** — A pencil of rays is any collection of rays other than a plane of rays.

10. **Relation of Rays.** — The sun is usually taken as the source of light, and, on account of its great distance from the earth, we may safely consider such solar rays as fall upon a terrestrial object of finite dimensions as parallel.

11. **Direction of Rays.** — The source of light may be assumed anywhere, but it is conventionally taken in such a position that the rays shall be parallel to that diagonal of a cube (the cube resting on *H* in the first quadrant with one face coincident with *V*) which slopes downward to the right toward *V*.

The horizontal and vertical projections of a ray will each make an angle of 45 degrees with *G-L*.

12. Visibility and Invisibility of Objects. — Unless otherwise stated, H and V will be considered opaque.

When projecting on H , the observer will be assumed to be above H , and objects above H will be considered visible and those below H invisible.

When projecting on V , the observer will be assumed to be in front of V , and objects in front of V will be considered visible and those behind V invisible.

13. Scale. — Unless otherwise stated the scale will be 1 inch to the unit.

CHAPTER II

NOTATION

14. To distinguish between a point in space and its projections on H and V , the point itself will be represented by the capital letter and its projection by the small letter.

15. The horizontal and vertical projections of a point, as M , will be represented by m , and m' respectively.

16. If a point, as M , be made to occupy several positions in the same problem, its horizontal and vertical projections will be represented in order by m , m' ; $m_{..}$, m'' ; $m_{...}$, m''' ; etc.

17. When two or more points are projected into the same point, the letters of all the points will be written upon this common projection.

18. If a point, as M , be revolved into H , its revolved position will be represented by m_H , and the vertical projection of the point in this revolved position by m'_H .

19. If a point, as M , be revolved into V , its revolved position will be represented by m^V , and the horizontal projection of the point in this revolved position by m'^V .

20. If a point, as M , be projected upon a plane, as T , other than H or V , this projection will be represented by MT , and the horizontal and vertical projections of this projection by mt , and mt' respectively.

21. If a point, as M , be projected upon a plane, as P , and this projection be projected upon another plane, as Q , the last projection will be represented by MPQ , and its horizontal and vertical projections by mpq , and mpq' respectively.

22. If in the last case MPQ were revolved into H , its revolved position would be represented by mpq_H ; if revolved into V , its revolved position would be represented by mpq^V .

23. A new ground line will be represented by either $G-L$, or $G'-L'$, according as it lies in the original H or V .

24. The shadow of a point, as M , upon H will be represented by m .

25. The shadow of a point, as M , upon V will be represented by m' .

26. The shadow of a point, as M , upon a plane other than H or V will be represented by MS , and the horizontal and vertical projections of this shadow by ms , and ms' respectively.

27. If in the last case the shadow MS were projected upon another plane, as P , its projection on this plane would be represented by MSP , and the horizontal and vertical projections of this projection by msp , and msp' respectively. If the point MSP were revolved into H , its revolved position would be represented by msp_H ; if into V , by msp^V .

28. The primary, secondary, tertiary, etc., shadows of a point, as M , will be represented by $MS1$, $MS2$, $MS3$, etc., and their horizontal and vertical projections by $ms1$, $ms1'$; $ms2$, $ms2'$; $ms3$, $ms3'$; etc.

29. The projections of given or required lines, if visible, will be represented by full lines.

30. The projections of given or required lines, when invisible, and the projections of all construction lines will be represented by broken lines, consisting of dashes about $\frac{1}{2}$ inch in length, with very small spaces between them, thus :

31. The traces of given or required planes, if visible, will be represented by full lines.

32. The traces of given or required planes, when invisible, and the traces of all construction planes will be represented by mixed lines, thus:

33. Lines connecting the projections of a point, and the projections of the path in which a point moves, will be represented by dotted lines, thus:

34. Visible shade and shadow lines will in general be represented by full heavy lines.

35. Invisible shade and shadow lines will in general be represented by broken heavy lines.

36. Shade lines which occur upon surfaces of single and double curvature will always be represented by light lines.

CHAPTER III

METHOD OF LOCATING GIVEN PARTS

37. Coördinate Planes of Projection. — Conceive a profile plane of projection, P , perpendicular to $G-L$, passing through the center of the drawing space and cutting V in a line known as the axis of Y .

All distances in space will be referred to the planes P , V , and H , and measured on perpendiculars from them.

Distances measured to the right of P , in front of V and above H , will be considered plus distances.

Distances measured to the left of P , back of V and below H , will be considered minus distances.

38. The Point. — The distance of a point from P is equal to the distance of its horizontal or vertical projection from Y . The distance of a point from V is equal to the distance of its horizontal projection from $G-L$. The distance of a point from H is equal to the distance of its vertical projection from $G-L$.

In giving the position of a point in space, its distance from P will be mentioned first, its distance from V second, and its distance from H last. $M=2, 5, 4$ means that the point M is two units to the right of P , five units in front of V , and four units above H ; that the horizontal projection of M is two units to the right of Y and five units in front of $G-L$; that the vertical projection of M is two units to the right of Y and four units above $G-L$.

The letters M , N , O , P , Q , and R will be used in connection with points.

39. The Line. — A line will be located by two of its points. $[M = -2, 2, -5; N = 2, -4, 6]$ indicates a line passing through the points M and N but not necessarily limited by them.

Lines will be specified by the letters of the points used in locating them.

40. The Plane. — Planes will be located by their horizontal and vertical traces, that is, by their intersections with H and V respectively.

The traces will be located by giving the position of their vertex, that is, their intersection on $G-L$, and the angles which they make with $G-L$.

The angles which the horizontal and vertical traces make with $G-L$ will always be measured, the former clockwise, the latter contra-clockwise, starting from $G-L$ on the right of the vertex.

Of the two angles made with $G-L$, that made by the horizontal trace will be mentioned first. $T = 4, 30^\circ, 45^\circ$ means that the vertex is four units to the right of Y ; that the horizontal trace runs downward toward the right, making an angle of 30 degrees with $G-L$; that the vertical trace runs upward toward the right, making an angle of 45 degrees with $G-L$. $S = 0^\circ, 4, 3$ indicates that the traces are parallel to $G-L$, that the horizontal trace is four units below $G-L$, that the vertical trace is three units above $G-L$.

Planes will be specified by the letters S , T , U , and W ; their horizontal traces by $S-s$, $T-t$, $U-u$, $W-w$; and their vertical traces by $S-s'$, $T-t'$, $U-u'$, and $W-w'$, the capital letter being placed at the vertex.

41. The Ground Line. — In general $G-L$ may have an arbitrary location upon the working plane. For plate work, however, the position of $G-L$ will be located with reference to the lower base of the problem space. $G-L = 2\frac{1}{4}$ is to be interpreted that $G-L$ is $2\frac{1}{4}$ units above the lower base of the problem space, and parallel to it.

CHAPTER IV

GENERAL DIRECTION IN REGARD TO THE SOLUTION OF PROBLEMS

42. *Shades and shadows* is an application of descriptive geometry, and its problems will be treated as such.

43. *The Solution of Problems Relative to the "Point," Fundamental in the Solution of all Problems.*—As a solid is determined by surfaces, a surface by lines, and a line by points, the method of finding the shadow of a point is the method fundamental in the solution of all the problems of the subject.

44. *To find the Shadow upon H of a Point in Space.*—Since the umbra of a point is the ray passing through the point, draw through the point a ray and find where the ray pierces H .

45. *To find the Shadow upon V of a Point in Space.*—Draw through the point a ray and find where the ray pierces V .

46. *To find the Shadow upon Any Surface of a Point in Space.*—Draw through the point a ray and find where the ray pierces the surface.

47. *To find the Shadow upon Any Surface of a Straight Line in Space.*—Since the umbra of a line (unless the line is a ray) is the plane of rays containing the line, pass a plane of rays through the line and find where the plane of rays intersects the surface.

48. *To find the Shadow upon Any Surface of a Plane Surface.*—Find the shadows of the lines which limit the plane surface.

49. *To find the Shade of a Solid.*—Determine the shade lines and then project them in heavy lines upon H and V .

7.

50. **To find the Shadow of a Solid upon Any Surface.** — Find the shadows of the shade lines upon the surface.

51. **Cross-hatching.** — Portions of surfaces in shade or shadow may be cross-hatched or tinted, but this is hardly necessary unless stress is to be laid upon the art side of the subject.

CHAPTER V

SHADOWS OF POINTS AND STRAIGHT LINES

52. Problem 1. — *Find the shadow of a point, M, upon H and V.*

See Plate I, Fig. 1. A ray through M pierces H at m_s , and V at m^s .

53. Problem 2. — *Find the horizontal and vertical projections of the shadow of a point, M, upon an oblique plane, T.* See Plate I, Fig. 2. A ray through M pierces the plane T at MS , horizontally and vertically projected at ms , and ms' respectively.

NOTE. — In Problem 2 assume the plane T in various positions, as: (a) perpendicular to H , (b) perpendicular to V , (c) perpendicular to $G-L$, (d) parallel to H , (e) parallel to V , (f) parallel to $G-L$, etc.

54. Problem 3. — *Find the shadow of a line, M-N, upon H and V.* See Plate I, Fig. 3. The shadow of M upon H is m_s ; the shadow of N upon H is n_s . Therefore the line m_s-n_s is the shadow of the line $M-N$ upon H . Proceed in the same way to find the shadow upon V .

55. Problem 4. — *Find the primary shadow of a line, M-N, upon H and V.* See Plate I, Fig. 4. The shadow of the line $M-N$ upon H is the line m_s-n_s and the shadow of the line $M-N$ upon V is the line m^s-n^s , the two lines intersecting $G-L$ at the common point o_s^s . The primary shadow is $n_s-o_s^s-m^s$.

NOTE. — To find the primary shadow of a line upon any two intersecting planes, find the shadow of the line upon each plane independently of the other, and retain the primary portion.

56. Problem 5. — *Find the horizontal and vertical projections of the shadow of a line upon an oblique plane.* See Plate I, Fig. 5.

Let T be the plane and $M-N$ the line. A ray through M pierces T at MS ; a ray through N pierces T at NS . Therefore the projections of the shadow of the line are the lines $ms, -ns$, and $ms' - ns'$.

NOTE.—In Problem 5 assume the plane in various positions, as: (a) perpendicular to H , (b) perpendicular to V , (c) parallel to H , (d) parallel to V , (e) parallel to $G-L$, etc.

57. Problem 6.—*Find the shadow of one line upon another.* See Plate I, Fig. 6. The line $m_s - n_s$ is the shadow upon H of the line $M-N$. The line $o_s - p_s$ is the shadow upon H of the line $O-P$. These two shadows intersect at q_s .

If from any point in the shadow of a line a ray be drawn, it must remain in the plane of rays which determines this shadow, and therefore intersect the line which casts the shadow. A ray through the point q_s (the point common to both shadows) must intersect both lines, $M-N$ and $O-P$, which it does at QS and Q respectively. Therefore QS is the shadow of the line $O-P$ upon the line $M-N$.

NOTE.—The problem may be solved by passing a plane of rays through the line $O-P$ and finding its intersection with the line $M-N$.

58. Problem 7.—*Determine whether a given plane is a plane of rays.* If through any point of a plane of rays a ray be drawn, it must remain in the plane and pierce H in the plane's horizontal trace and pierce V in the plane's vertical trace. See Plate I, Fig. 7. The point M is assumed in the plane T . A ray through M pierces H at m_s outside of the trace $T-t$, and pierces V in m' outside of the trace $T-t'$. Therefore the plane T is not a plane of rays.

59. Problem 8.—*Find the shadow upon H of a square card whose surface is parallel to H .* See Plate I, Fig. 8. $A-B-C-D$ represents the card; a_s, b_s, c_s , and d_s are the shadows upon H of A, B, C , and D respectively. Therefore $a_s - b_s - c_s - d_s$ is the shadow required.

60. Problem 9.—*Given a square card whose surface is parallel to H and above it; required the horizontal projection of the shadow cast by the card upon a plane parallel to H and above it.* See Plate I, Fig. 9. Let the vertical trace of the horizontal plane be $G'-L'$, which in one sense is a new ground line. Let $A-B-C-D$ represent the card. A ray through the point A pierces the new plane at AS , horizontally projected at as .

In the same way bs , cs , and ds , are found. $as, -bs, -cs, -ds$, is the horizontal projection required.

NOTE.—This rule applies to any horizontal plane, above or below H .

61. Problem 10.—*Given a square card whose surface is parallel to H ; required the horizontal projection of the shadow cast by the card upon a plane perpendicular to V , but oblique to H .*

62. Problem 11.—*Find the shadow upon V of a square card whose surface is parallel to V .* Proceed as in Problem 8.

63. Problem 12.—*Given a square card whose surface is parallel to V and in front of it; required the vertical projection of the shadow cast by the card upon a plane parallel to V and back of it.* See Plate II, Fig. 10. Let the horizontal trace of the plane back of V be $G,-L,,$ which in a sense is a new ground line. Let $A-B-C-D$ represent the card. A ray through the point A pierces the new plane at AS , which is vertically projected at as' . In the same way bs' , cs' , and ds' are found, and $as'-bs'-cs'-ds'$ is the vertical projection required.

NOTE.—This rule applies to any plane parallel to V .

64. Problem 13.—*Given a square card whose surface is parallel to V ; required the vertical projection of the shadow cast on a plane perpendicular to H , but oblique to V .*

65. Problem 14.—*Find the horizontal and vertical projections of the shadow cast by a square card upon a plane parallel to $G-L$.*

66. Problem 15.—*Find the primary shadow cast upon H and V by a square card whose surface is perpendicular to H , but oblique*

to V . See Plate II, Fig. 11. $A-B-C-D$ is the card; $a_s-b_s-c_s-d_s$ is the shadow of the card upon H ; $a^s-b^s-c^s-d^s$ is the shadow of the card upon V ; $a_s-e_s-f_s-d_s$ is the primary portion of the shadow on H ; $e_s-b^s-c^s-f_s^s$ is the primary portion of the shadow on V . Therefore $a_s-e_s-b^s-c^s-f_s^s-d_s$ is the required shadow.

Principle. — See Plate II, Fig. 11. When a line, as $C-D$, is perpendicular to H , its shadow on H is a straight line parallel to the horizontal projection of a ray, and its shadow on V is a straight line perpendicular to $G-L$. *Prove.*

Principle. — See Plate II, Fig. 11. The shadow of a line, as $A-D$, upon a plane, as H , to which the line is parallel, is a line equal and parallel to the line itself. *Prove.*

Principle. — See Plate II, Fig. 11. If any number of lines, as $A-D$, $B-C$, etc., are parallel, their shadows upon any plane, as H , are also parallel. *Prove.*

67. Problem 16. — *Find the primary shadow upon H and V of an hexagonal card whose surface is perpendicular to H , but oblique to V .* See Plate II, Fig. 12. Let $A-B-C-D-E-F$ represent the card. The primary shadows of the six vertexes are a_s, b_s, c^s, d^s, e^s , and f_s . Therefore $a_s-b_s-g_s^s-c^s-d^s-e^s-k_s^s-f_s$ is the required shadow.

68. Problem 17. — *Find the primary shadow upon H and V of a circular card whose surface is parallel to H .* See Plate II, Fig. 13. Let $A-B-D-E$ represent the card. The shadow of the circle upon H is a circle whose center is the shadow of the center of the given circle, and whose radius is the radius of the given circle. *Prove.*

The shadow of the circle upon V will be an ellipse, which may be determined by assuming a sufficient number of points upon the circumference of the circle and finding their shadows on V .

The point F , for example, is assumed on the circumference and its shadow, f^s , found on V . The limits of the arc which casts the shadow upon V are determined by drawing rays backward from a_s^s and g_s^s until they intersect the arc.

CHAPTER VI

SOLIDS WITH RECTILINEAR OUTLINES

NOTE.—Hereafter, unless otherwise stated, the primary shadow alone will be required.

69. Problem 18.—*Find the shadow upon H and V of a cube, two of whose faces are parallel to H.* See Sections 50, 7 and 6. See Plate II, Fig. 14. The shade lines in this case are $A-B$, $B-C$, $C-D$, $D-E$, $E-F$, and $F-A$. The shadow of $A-B$ is a_s-b_s ; the shadow of $B-C$, $b_s-o_s'-c_s$, etc. Portions of the shadows of $C-D$ and $D-E$, on V , fall behind the cube and are therefore represented by broken lines. Portions of the shadows of $E-F$ and $F-A$, on H , fall below the cube and are therefore represented by broken lines.

Note that the shade lines are projected in heavy lines. The visible face, $A-B-C-G$, is in shade and for this reason cross-hatched.

70. Problem 19.—*Find the shadow upon H and V of an hexagonal prism whose bases are parallel to H.* See Plate III, Fig. 15. The shade lines in this case are $A-B$, $B-C$, $C-D$, $D-E$, $E-F$, $F-G$, $G-K$, and $K-A$. The shadow of $A-B$ is $a_s-o_s'-b_s$, the shadow of $B-C$ is b_s-c_s , etc.

71. Problem 20.—*Two horizontal cantilevers extend from a vertical wall; required the shadow of each upon the vertical wall, and the shadow of the upper cantilever on the upper surface of the lower.* See Plate II, Fig. 16.

Let $A-B-C-D-E$ represent a cantilever extending toward the right, and $F-G-H-K-L$ another extending toward the left and

below the former. The plane V represents the vertical wall. The shadow of the first cantilever upon V is $a'-b'-c'-d'-e'$.

To find the shadow of the upper cantilever upon the upper surface of the lower; find the shadow of the upper cantilever upon a plane containing the upper surface of the lower, and retain that portion of the shadow which falls upon the cantilever. The vertical trace of this auxiliary plane is $G'-L'$.

72. Problem 21. — *Given a flight of steps and a horizontal girder resting on the top; required the shadow of the girder on H , and also on the rise and tread of each step. See Plate III, Fig. 17. $A-B$ and $C-D$ are shade lines of the girder.*

$G,-L$, is the horizontal trace of the plane of the rise of the first step. $BS2$ is the shadow of B upon this plane. $CS2$ is the shadow of C upon the same plane. $A-B$ intersects this same plane at F , and $C-D$ intersects it at E . $F-BS2$ is the shadow of $A-B$ upon the plane of the rise of the first step.

For the same reason $E-CS2$ is the shadow of $C-D$ upon the same plane. The shadow of the girder, upon the rise in question, will be included between these last two lines.

$G'-L'$ is the vertical trace of the plane of the tread of the first step. $BS1$ is the shadow of B upon this plane, and $CS1$ is the shadow of C upon the same plane. Lines through these points parallel to $A-B$ and $C-D$ will be the shadows of these lines upon the plane.

73. Problem 22. — *Given a square pillar surmounted by an hexagonal abacus; required the shadow of the abacus upon the pillar and the shadow of both upon H and V . See Plate III, Fig. 18.*

The shade lines on the abacus are $A-B$, $B-D$, $D-G$, $G-H$, $H-K$, $K-L$, $L-M$, and $M-A$.

The shadow of the point A upon the plane of the front face of the pillar is AS .

The point in $A-B$ casting shadow on the edge $V-W$ is F . *Prove.* Its shadow is FS . $AS-FS$ is the shadow of $A-B$ on the front face of the pillar.

BS is the shadow of B on the left face of the pillar. The point in $B-D$ which casts shadow on the edge $U-Q$ is E . *Prove.* Its shadow is ES .

The line $n,-o,$ is the shadow of $N-O$ on H . The line $u,-q,$ is the shadow of $U-Q$ on H .

74. Problem 23. — *Given a roof, and a chimney with cornice; required the shadow of the cornice on the chimney and the shadow of both on the roof.* See Plate III, Fig. 19. Let $G-N-M-K-O$ represent the roof. The shade lines of the cornice are $A-B, B-C, C-D, D-E, E-F$, and $F-A$.

Project the important points of the roof and chimney upon the profile plane P , and revolve this plane about its vertical trace until all fall into V . The point ap^V is the revolved position of the profile projection of A ; and the line ap^V-asp^V is the revolved position of the profile projection of a ray through A . The point asp^V is the revolved position of the profile projection of the shadow of A on the roof. The point $asp,$ is the horizontal projection of the profile projection of the shadow of A on the roof, and a line through $asp,$ parallel to $G-L$ must contain the horizontal projection of the shadow of A on the roof. *Prove.* A ray through A must also contain the shadow of A , and therefore AS is the shadow sought.

This method is often employed when the planes receiving the shadows are parallel to $G-L$.

75. Problem 24. — *Find the shadow cast upon a block of steps by an upright which supports a horizontal girder.* See Plate IV, Fig. 20. $A-B$ and $D-E$ are the important shade lines of the girder, and $G-H$ and $L-K$ the important shade lines of the upright.

The surfaces on which the shadows are cast are either horizontal or vertical. Shadows cast on horizontal surfaces will be

represented in horizontal projection and those cast on vertical surfaces will be represented in vertical projection. As the shade lines of the girder are horizontal, their shadows upon horizontal surfaces will be parallel to the lines themselves, and one point in the shadow of each will be sufficient.

The vertical trace of the plane of the upper surface of the block is $G'''-L'''$. The shadows of the two shade lines $A-B$ and $D-E$ upon this surface are $AS1-BS1$ and $DS1-ES1$ respectively, giving the limits of the shadow of the girder upon this surface.

The vertical trace of the plane of the tread of the second step is $G''-L''$. The shadows of B and D upon this plane are $BS2$ and $DS2$ respectively, and lines drawn through these points parallel to $A-B$ and $D-E$ will limit the shadow of the girder on this plane. For the same reason, lines drawn through $BS3$ and $DS3$ parallel to $A-B$ and $D-E$ will limit the shadow of the girder upon the tread of the first step.

The shadow of the left wall upon the steps will be limited by the shadows of the shade lines $C-F$ and $F-M$. The shadow of $F-M$ on the plane of the tread of the second step is a line through $FS1$ parallel to $F-M$. The shadow of $F-M$ on the plane of the tread of the first step will be a line through $FS2$ parallel to $F-M$. $C-FS2$ is the shadow of $C-F$ upon this same plane.

The horizontal trace of the plane of the left face of the block is $G_{II}-L_{II}$. The shadow of the upright upon this face will be included within the intersections with this face of planes of rays through $G-H$ and $K-L$. $A-B$ also casts a small shadow upon the same face determined by the two points $AS2$ and $BS4$.

The horizontal trace of the plane of the rise of the third step is $G_{III}-L_{III}$. Two points in the shadow of $E-D$ upon this plane are P and Q , previously found. In this way the shadows of $A-B$ and $D-E$ upon the rises of the other steps may be found, as may be seen from the diagram.

The horizontal trace of the plane of the left face of the right wall is G_v-L_v . The shadows of D and E upon this plane are $DS4$ and $ES4$ respectively.

The horizontal trace of the plane of the front face of the block is G_v-L_v and the shadow of $E-D$ upon this plane may be found as in previous cases.

CHAPTER VII

SOLIDS WITH RECTILINEAR AND CURVILINEAR OUTLINES

76. The Determination of Shadows which are cast by Plane Curves upon Curved Surfaces.—It is often required to find upon the interior of such surfaces as cones, cylinders, etc., the shadow cast by some plane section of the surface.

For example: Given a hollow cylinder whose interior is exposed to the light; required the shadow of the upper base, which is a plane section of the cylinder, upon the interior surface.

It may be proved by analytic geometry that such shadows are plane curves.

When these curves are ellipses, their axes or diameters may be determined and the curve constructed by reference to them, as will be explained later in this chapter.

In the majority of cases, however, shadows of this character may be easily and accurately determined by finding the shadows of a sufficient number of points selected upon the curves casting the shadows.

77. Problem 25.—*Given a right circular cylinder with axis perpendicular to H; required the shade elements and the primary shadow upon H and V.* See Plate III, Fig. 21. Draw two planes of rays, S and T, tangent to the cylinder. The elements of tangency, E-F and G-K, are the shade elements, since they separate the light portions of the surface from the dark. The arcs F-B-D-K and G-L-E complete the shade line of the body.

The shadow of the arc E-L-G upon H is an equal arc with c_s as a center. The shadow of the arc F-B-D-K upon H is

an equal arc with a_s as a center. The shadow of this same arc upon V is an ellipse, best determined by finding the shadows of a sufficient number of points of the arc.

The portion of the surface of the cylinder between the element $E-F$ and the extreme element on the right will be in visible shade.

In all the problems of the preceding chapter, lines have been such as were determined by the intersections of surfaces—in nearly every case plane surfaces. In representing the projections of single and double curved surfaces, lines are referred to which are not intersections of surfaces but limits of visible portions of surface. For example, the outline of the visible portion of a sphere is a circle.

On such surfaces there are shade lines, boundaries between light and dark portions, which come under this class and should never be drawn as heavy lines.

78. Problem 26.—*Given a conical pillar with an hexagonal abacus; required the shade of the abacus, the shade of the pillar, and the vertical projection of the shadow of the abacus on the pillar. See Plate IV, Fig. 22.*

The vertex of the cone is E , its shadow on H is e_s . The horizontal trace of every plane of rays containing E must pass through e_s . A ray through B pierces H at b_s , therefore b_s-e_s is the horizontal trace of a plane of rays containing B and E . The ray through B must remain in this plane and intersect the surface of the cone in the element cut by the plane. This element is $E-D$, and where it is cut by the ray will be the shadow of B on the surface of the cone. This shadow is vertically projected at bs' . In the same way the shadows of any number of points of the shade lines $A-B$ and $B-C$ may be found.

The line e_s-f , is the horizontal trace of a plane of rays tangent to the cone along the shade element $E-F$.

79. Problem 27.—*Given an hexagonal pillar with a cylindrical abacus; required the shade lines on the abacus and pillar, and the shadow of the abacus on the pillar.* See Plate II, Fig. 23.

The shade elements on the abacus are $A-B$ and $C-D$. *Prove.* The lower boundary of the shadow of the abacus on the pillar will be the shadow of the shade line $D-G-F-E-B$ upon the face of the pillar. The shadow of E is ES , the shadow of F is FS , etc.

80. The Ellipse.—Any chord through the center of an ellipse is a diameter of the ellipse. That diameter of an ellipse which contains the foci is the major or transverse axis. That diameter of an ellipse which is perpendicular to the transverse axis is the minor or conjugate axis.

The major axis of an ellipse is the longest diameter of the ellipse and the minor axis is the shortest diameter of the ellipse.

Two diameters of an ellipse are conjugate when each of the two is parallel to the linear tangents to the ellipse at the extremities of the other.

The shadow of a circle upon a plane to which its surface is not parallel is in general an ellipse. The shadow of the center of the circle will be the center of the ellipse, and the shadow of any diameter of the circle will be a diameter of the ellipse.

The shadows of two perpendicular diameters of a circle will be conjugate diameters of the ellipse of shadow. *Prove.*

81. Problem 28.—*Given the conjugate diameters of an ellipse; required the axes.* See Plate II, Fig. 24. The lines $a,-c,$ and $d,-b,$ are the given conjugate diameters.

Draw $G-L$ through c , parallel to $d,-b,$; also $c,-a'$ perpendicular to $G-L$. Make $c'-c$, equal to $d,-c,$ and draw the circle $a'-b'-c'-d'$. Now the ellipse whose conjugate diameters are $c,-a,$ and $d,-b,$ may be regarded as the shadow of this circle, since $d,-b,$ may be regarded as the shadow of $d'-b',$ $c,-a,$ as the shadow of $c,-a',$ and upon two conjugate diameters one, and

only one, ellipse can be constructed. Produce $a'-c$, to l ,, making $c-l$, equal to $c'-c$,. Connect l , and c ,, and at the middle point of this line draw a perpendicular cutting $G-L$ in m' . With m' as a center and a radius equal to $m'-c'$ describe a circle through c' , c , and l ,, cutting $G-L$ at n' and o' . Draw $n'-c'$ and $o'-c'$, cutting the circumference of the circle whose center is c' at e' and f' respectively. These lines will be perpendicular to each other and their shadows will fall on $n'-c$, and $o'-c$, respectively. The shadow of e' is e , and the shadow of f' is f . Therefore $e-c$, and $f-c$, will be the semiaxes of the ellipse, for they are semidiameters, and they are perpendicular to each other. *Prove.*

82. Problem 29. — *To draw the ellipse directly from the conjugate diameters.* Trammel method. See Plate IV, Fig. 25. Let $A-B$ be the major conjugate diameter and $D-E$ the minor conjugate diameter. Through E draw $G-F$ perpendicular to $A-B$ and lay off $E-G$ equal to $A-C$. Connect G and C by a straight line and produce. $A-B$ and $G-K$ are the *trammel axes*, $F-G$ the *trammel length*, and $E-F$ and $E-G$ the *trammel divisions*.

The ends of the trammel are now kept coincident with their respective axes and the point E traces the ellipse. $F-G$ is one position of the trammel and E one point on the ellipse. $F'-G'$ is another position of the trammel and E' another point of the ellipse.

83. Problem 30. — *Given a hollow oblique cylinder of thin material; find the shade of the cylinder, the shadow of the cylinder on H and V , and the shadow of the upper base on the interior surface.* See Plate IV, Fig. 26.

The shadow of B on H is b ,. The line $c,-b$, is the horizontal trace of a plane of rays through $C-B$. The planes of rays tangent to this cylinder must be parallel to the one just found, and the lines $a,-d$, and $m,-l$, will be their horizontal traces. $M-L$ and $A-D$ are the elements of tangency, and therefore the

elements of shade. The shade lines are then $A-D$, $D-E-L$, $L-M$, and $M-G-A$.

The shadows on H of the shade lines $A-D$ and $L-M$ will be $a,-d,$ and $m,-l,$ respectively, previously found. The shadow on H of the arc $D-E-L$ will be an equal arc with $b,$ as a center.

To find the shadow on V : Assume any point, as $E,$ on the shade line $D-E-L,$ and find its shadow, $e^s.$ Assume other points and proceed in the same way. The shadow on V of D is $d^s,$ and the shadow of the shade line $D-A$ is easily found.

To find the shadow of the upper circle on the interior surface: Assume any point, as $F,$ on the circumference of the circle. Now a ray through F must remain in a plane of rays through F and intersect the surface of the cylinder where the plane of rays intersects the surface.

Therefore through $F,$ and for convenience parallel to the axis of the cylinder also, draw a plane of rays. The point $g,$ is the point in which the element through F pierces $H.$ The line $g,-n,$ is the horizontal trace of the plane of rays through this element. $N-K$ is the element which this plane cuts from the opposite side of the cylinder.

Now draw a ray through F and find the point FS in which this ray intersects $N-K.$ FS is the shadow of one point, and others may be found in the same way.

84. Problem 31. — *Given a conical empty tank; find the shade lines, the shadow of the tank on H and V , and the shadow of the upper interior circle on the interior surface. See Plate V, Fig. 27.*

The vertex of the outer cone is at $A,$ the vertex of the inner at $D.$ A ray through A cuts H at $a,$ and one through D cuts H at $d.$ Horizontal traces of planes of rays through the vertexes A and D must pass through $a,$ and $d,$ respectively.

The shade lines $A-F$ and $A-E$ on the outer surface are the elements of the cone along which planes of rays are tangent.

The shadows of the upper circles on H will be equal circles with centers at c_s .

The shadow of the outer circle upon V will be an ellipse, which may be found by casting the shadows of points assumed on the circle. For example, the shadow of G is g'' .

To find the shadow of the upper interior circle upon the interior surface: Assume any point, as L , on the inner circle. Now, as in Section 83, a ray through L must remain in a plane of rays through L and intersect the surface of the cone where the plane of rays intersects the surface. Therefore through L , and for convenience also through the vertex of the cone, draw a plane of rays. The point s , is the point in which the element through L pierces H . The line d_s-s-m , is the horizontal trace of the plane of rays through this element. $D-Q$ is the element which this plane cuts from the opposite side of the cone. Now draw a ray through L and find the point LS in which this ray intersects $D-Q$. LS is the shadow of one point, and others may be found in the same way.

85. Problem 32. — *Given an ellipsoid of revolution; required the shade of the ellipsoid and the shadow of the ellipsoid on H . See Plate V, Fig. 28.*

The shade line is the locus of the points at which rays are tangent to the ellipsoid, or it is the line of contact of a cylinder of rays tangent to the ellipsoid, and is an ellipse.

$C-c_s$ is the axis of this cylinder. A plane through $C-c_s$ perpendicular to H is a meridian plane both of the cylinder and of the ellipsoid, cutting from the latter an ellipse and from the former two elements tangent to the ellipse.

These points of tangency will be the highest and lowest points in the ellipse of shade, and the extremities of its long axis. The short axis of the same ellipse will pass through the center of the long axis and be perpendicular to the assumed meridian plane.

Revolve this meridian plane about a vertical axis through C until the plane is parallel to V ; then the vertical projection of the meridian curve will coincide with the vertical projection of the ellipsoid. The line $c'-c'_{ss}$ will be the vertical projection of the axis of the cylinder in revolved position. The lines $a''-d''$ and $b''-e''$, parallel to $c'-c'_{ss}$ and tangent to the vertical projection of the meridian ellipse in revolved position, will be the vertical projections, in revolved position, of the highest and lowest elements of the cylinder, and will determine the two points of tangency A and B . Therefore $A-B$ is the long axis and $F-G$ the short axis of the ellipse of shade.

The horizontal projections of these axes will be the axes of the horizontal projection of the ellipse of shade. The vertical projections of these axes will be the conjugate diameters of the ellipse of shade.

Points of the ellipse of shade may be found thus: Pass planes, as S , perpendicular to the axis of the ellipsoid. The plane S cuts from the surface of the ellipsoid the circle $L-M-N$, from the axis $A-B$ the point K , and from the plane of the ellipse of shade the line $L-M$ parallel to $F-G$ and through K . The intersections, namely, L and M , of the line $L-M$ with the circle $L-M-N$ will be two points of the ellipse of shade.

The shadow of the ellipsoid upon H will be the shadow of the ellipse of shade, or it will be the ellipse constructed upon the shadows of the axes $A-B$ and $F-G$.

86. Problem 33. — *Find the shade of a sphere and the shadow of the sphere on H and V .* Apply the principles of the preceding problem.

This problem may be solved by passing a series of vertical planes of rays, cutting from the sphere circles, and from the tangent cylinder of rays elements tangent to these circles at points of the curve of shade. These points may be found by

revolving the cutting planes with their contents about their horizontal traces into H .

87. Problem 34.—*Find the shadow of the upper circle of a hollow hemispherical shell upon the inner surface.* See Plate V, Fig. 29.

The line $a,-c_s$ is the horizontal trace of a vertical plane of rays through the center. This plane cuts from the hemisphere a semicircle which intersects the upper circle at A .

Now if we pass through A a ray and revolve the ray and the semicircle about the horizontal trace of the plane of rays into H , they will take the positions shown in the diagram. The point as_H is the revolved position of the shadow of A on the surface, and when revolved back to true position will be projected at as , and as' .

By passing other vertical planes of rays, as the one whose horizontal trace is $d,-g,,$ and revolving them about their own traces we shall obtain other points of the shadow.

88. Problem 35.—*Find the shadow of the edge of a niche on the interior surface.* First method. See Plate V, Fig. 30.

The niche consists of a semicylinder surmounted by a half hemisphere. The horizontal projection is shown in the semicircle $a,-as,-o,-a,,$

The shade lines are $B-A$ and $A-D-E-F-K-G.$ The shadow of $A-B$ on the interior surface is $AS-M.$ The shadow of $A-F-G$ falls partly on the cylinder and partly on the sphere.

To find the portion of the shadow on the cylinder: Assume a point, as D , and through it draw a ray piercing the surface at $DS.$ DS is the shadow of $D.$ Other points of the shadow on the cylinder may be found in the same way.

To find the portion of the shadow on the sphere: Through any point, as K , draw a ray and through this ray draw a plane of rays perpendicular to $V.$ Revolve this plane about its vertical trace $k'-o'$ into $V.$ The semicircle cut from the surface takes

the position $k'-ks^V-o'$. Any point, as N , in the ray through K will fall at n^V and the ray itself at $k'-n^V$. The line $k'-n^V$ intersects the revolved position of the semicircle at ks^V . KS is the shadow of K on the surface. The shadows of other points may be found in the same way.

89. Principle. — In finding the shadow of a line, straight or curved, upon a surface of single or double curvature, if we cast the shadow of the line upon an auxiliary plane which cuts the given surface in a line, the intersections of this line with the auxiliary shadow (provided there be such intersections) will be points in the shadow sought; for these points are both upon the surface where the shadow is to be cast and in the umbra of the line casting the shadow.

90. Problem 36. — *Find the shadow of the edge of a niche on the interior surface.* Second method. See Plate V, Fig. 31.

Apply the principle of Section 89. The line $a,-p,-b,-a$, is the horizontal projection of the niche and $a'-d'-f'-e'-b'$ is the vertical projection. As before, $ds\ 2'-p'$ is the vertical projection of the shadow of the edge $D-A$.

To find the shadow of $D-F-O$: Pass, parallel to V , a system of planes whose horizontal traces are $S-s_1$, $T-t_1$, $U-u_1$, etc. The plane U , for example, cuts from the niche the line $G-H-K-L-M$. The shadow of C on this plane is CS , and $ds\ 1'-ns'-es'$, drawn with es' as a center and $C-D$ as a radius, is the vertical projection of the shadow of $D-F-E$ on this plane. The point ns' , the intersection of the circle $ds\ 1'-ns'-es'$ with the circle $h'-k'-l'$, is the vertical projection of a point both in the surface of the niche and in the umbra of $D-F-E$, and therefore the vertical projection of the shadow of one point of the edge on the interior surface.

By using other planes parallel to V other points of the shadow may be found.

NOTE. — Professor Lawrence of the Massachusetts Institute of Technology has the credit of introducing this second method.

91. **Principle.** — If a plane of rays be drawn tangent to any surface of revolution, the point of tangency will be a point in the line of shade. The meridian plane of the surface containing this point of tangency will be perpendicular to the tangent plane, cut a meridian curve from the surface, and intersect the tangent plane in a line tangent to the meridian curve at the point in which the plane is tangent to the surface. Any plane projecting a ray upon this meridian plane will be perpendicular to the meridian plane and parallel to the tangent plane of rays. **Prove.** Therefore the intersection of the meridian and tangent plane must be parallel to the projection of a ray upon the meridian plane. If, then, we pass any meridian plane through a surface of revolution and draw a line tangent to the resulting meridian curve and parallel to the projection of a ray upon this meridian plane, the point of tangency must be a point in the curve of shade.

92. **Problem 37.** — *Find the shade line on the surface of a torus.* Apply the principle of Section 91. See Plate V, Fig. 32.

If a plane surface of the shape shown in the vertical projection of this figure be revolved about its vertical central axis, it will generate a *torus*. Pass through the surface any meridian plane, as $T-t_1$. Project any ray, as $C-CS$, upon this plane. The horizontal projection of this projection is $c'-cst'$, and the vertical projection is $c''-cst''$. If we now revolve the plane T about the vertical central axis until the plane becomes parallel to V , the meridian curve cut by the plane T will then be vertically projected into the vertical projection of the surface. The projection of the ray upon this plane will then be vertically projected in $c''-cst''$. Lines drawn parallel to $c''-cst''$ and tangent to the vertical projection of the torus give A and B as points of tangency. The points a'' and b'' will be the vertical projections of two points, in revolved position, of the line of shade. When the plane is revolved back, a_1 will take the

position a , and b , the position b ; a'' will take the position a' and b'' the position b' . Other points may be found in the same way by using other meridian planes.

93. *Problem 38.—To find the shade and the shadow of any surface of revolution.* Apply the principle of Section 91 in finding the shade line and then apply the principle of Section 89 in finding the shadow.

PART II

PERSPECTIVE

CHAPTER VIII

GENERAL PRINCIPLES AND DEFINITIONS

94. The Subject defined. — Perspective is that branch of drawing which has to do with methods of representing objects as they appear.

95. Appearance of Objects in Nature. — Objects in Nature do not appear as they really are; for example, if you stand between the rails of a straight stretch of railway track, the rails appear to converge to a point, yet they do not converge.

Again, if you stand by the side of a long building and observe a series of windows of the same height, the size of each window seems to vary directly as its distance from you.

This is due to the fact that when rays of light, which are practically straight lines, come from objects to the pupil of the eye, they must come in convergent lines.

In Plate VI, Fig. 33, let *C* represent the pupil of the eye, *R-r* the surface of the retina, *A-B* and *D-E* two objects of the same dimensions but at different distances from *C*, *A-a*, *B-b*, *D-d*, and *E-e* rays of light through the pupil *C*.

The image of *A-B* upon the surface *R-r* is *a-b*, while the image of *D-E* upon the same surface is *d-e*, considerably smaller than *a-b*.¹

¹ The principles set forth in this article are not optically exact, but they are near enough to the truth to prove the statement in hand.

96. Deductions. — If now, referring again to Plate VI, Fig. 33, we place a transparent screen or plane, $P-p$, between the objects and the eye and at the same distance from C as the surface $R-r$, we shall have upon this plane $P-p$ the same pictures as appeared inverted upon the surface $R-r$.

This at once suggests a method of procedure in making a perspective drawing.

Assume a station point, C , for the eye, and a transparent picture plane, $P-p$, between the object and the eye. Through all the important points of the object draw rays to the eye and note their intersection with the picture plane. These intersections will mark the outline of the perspective drawing.

The perspective, then, of a point is the intersection with the picture plane of a ray through the point and the eye, and is therefore a point.

The perspective of a point situated in the picture plane is the point itself.

The perspective of a line is the intersection with the picture plane of a plane through the line and the eye, and is in general a line.

97. Size of the Drawing. — It is evident from Plate VI, Fig. 33, that the size or scale of the drawing will depend upon the relative positions of the eye, the object, and the picture plane.

If the eye and object are fixed in position, the size of the picture will vary with the backward and forward movement of the plane.

If the eye and plane be fixed, the size will vary with the distance of the object from the eye, etc.

98. Perspective by Descriptive Geometry. — We can now show the relation between perspective and descriptive geometry, and how the rules of the latter may be employed in this work.

In Plate VI, Fig 34, let C be the station point of the eye, $A-B-D-E$ a rectangular card, and $P-L-G$ a transparent plane

placed between the card and the eye. If rays be drawn from C to A , B , D , and E , and their intersections with $P-L-G$, viz., a^p , b^p , d^p , and e^p , be joined, $a^p-b^p-d^p-e^p$ will be the perspective of the card.

Regarding $P-L-G$ as V and $Q-L-G$ as H , the orthographic projections of C will be at c , and c' , and the projections of the card at $a,-b$, and $a'-b'-d'-e'$. The projections of the ray through A and C will be $a,-c$, and $a'-c'$, and the intersection of this ray with V will, by descriptive geometry, be a^p .

Therefore if we know the position of the object, and the position of the eye with reference to H , V , and P , we can, by descriptive geometry, find the perspective of the object upon V .

99. Picture Plane.—The picture plane is the plane on which the perspective drawing is made, and in this work will be considered coincident with V .

100. Point of Sight.—The point of sight is the position occupied by the eye of the observer.

101. Center of Vision, or Center of the Picture, is the vertical projection of the point of sight.

102. Axis of Vision.—The axis of vision is the vertical projecting line of the point of sight.

103. Visual Ray.—A visual ray is any ray through the eye.

104. Visual Plane.—A visual plane is any plane through the eye.

105. System of Lines.—A system of lines is any set of parallel lines.

106. System of Planes.—A system of planes is any set of parallel planes.

107. Ground Line.—The ground line has the same meaning here as in descriptive geometry, and in the location of parts will be regarded as coincident with the axis of X .

108. Horizontal Line.—The horizontal line is a line through the center of vision parallel to the ground line.

109. **Method of Locating Given Parts.** — See Shades and Shadows, page 9.

110. Such conventions as have already been given in connection with descriptive geometry and shades and shadows are to be observed also in perspective.

111. The perspective of a point, as M , will be designated by m^p .

112. The point of sight will be designated by C .

113. The center of vision will be designated by c' .

114. Initial points will be designated by $ip\ 1$, $ip\ 2$, etc.

115. Vanishing points will be designated by $VP\ 1$, $VP\ 2$, etc.

116. Points of distance will be designated by $PD\ 1$, $PD\ 2$, etc.

117. Measuring points will be designated by $MP\ 1$, $MP\ 2$, etc.

118. The picture plane, or V , will be designated by V .

119. The horizontal line will be designated by $H-L$.

120. **Scale.** — Unless otherwise stated, the scale will be 1 inch to the unit.

121. **Problem 39.** — *Find the perspective of the point $M = -1, -\frac{1}{2}, 1\frac{1}{2}$, when $C = \frac{1}{2}, 2, \frac{3}{4}$.* See Plate VI, Fig. 35. The eye of the observer is projected at (c, c') and the point M at (m, m') . The line m_c , is the horizontal projection of the visual ray through M , and m'_c is the vertical projection of the same ray. Therefore m^p , the intersection of this visual ray with V , is the perspective required.

122. **Problem 40.** — *Find the perspective of the line [$M = 0, -\frac{1}{2}, 1\frac{1}{2}; N = 1\frac{1}{2}, -\frac{1}{2}, 1$], when $C = -1, 2, \frac{3}{4}$.* See Plate VI, Fig. 36. As in Problem 39, m^p must be the perspective of M , and n^p must be the perspective of N . Therefore the line m^p-n^p is the perspective sought.

123. **Problem 41.** — *Find the perspective of a rectangular card, $A-B-D-E$, situated in the second quadrant and having its surface perpendicular to H and oblique to V . $C = -1\frac{1}{2}, 2, 2\frac{1}{2}$.* See Plate VI, Fig. 37.

The points a^p , b^p , d^p , and e^p are the perspectives respectively of A , B , D , and E . Therefore the figure $a^p-b^p-d^p-e^p$ is the perspective sought.

124. Problem 42. — *Find the perspective of a cube, $A-B-D-E-F-G-K-L$, resting on H and somewhat back of V . $C = 1\frac{3}{4}$, $2\frac{3}{4}$, $2\frac{1}{2}$. See Plate VI, Fig. 38. The points a^p , b^p , d^p , e^p , f^p , g^p , k^p , and l^p are the perspectives respectively of A , B , D , E , F , G , K , and L . Therefore the figure $a^p-b^p-d^p-e^p-f^p-g^p-k^p-l^p$ is the perspective sought.*

125. Problem 43. — *Find the perspective of a $1\frac{1}{2}$ inch square card whose surface is parallel to H and whose edges are oblique to V . Center of card = $1, -2, \frac{1}{2}$; $C = -1\frac{1}{2}, 3, 2$.*

126. Problem 44. — *Find the perspective of an hexagonal card whose surface is in H . Center of card = $-1, -2, 0$; $C = 1\frac{1}{2}, 3, 2$.*

127. Problem 45. — *Find the perspective of a square prism resting on H , 2 back of V and 2 to the left of P . $C = \frac{1}{2}, 4, 1$.*

128. Problem 46. — *Find the perspective of a cross whose arm is perpendicular to V and whose upright rests on H , 2 back of V and 1 to the right of P . $C = -2, 4, 1$.*

129. Perspective by Aid of Vanishing Points. — It is evident that the perspective drawing of any object can be made when we have the projections of the object and the projections of the point of sight, but the processes thus far explained are long and laborious.

If the line $M-N$ in Plate VI, Fig. 39, be produced, it will intersect V at a' . The point a' is at the same time in $M-N$ and in V and is therefore one point in the perspective of $M-N$ (see Section 96).

Again, if through C we draw a line, $C-VP\ 1$, parallel to $M-N$, it must remain in the visual plane of $M-N$ and intersect V somewhere in the line in which the visual plane intersects V .

The point $VP\ 1$, the point in which the line $C-VP\ 1$ intersects V , is therefore the perspective of another point in $M-N$,

and the line $a'-VP\ 1$ is the indefinite perspective of the line $M-N$.

This is verified by the fact that the line $a'-VP\ 1$ passes through the perspectives of the two points M and N .

The point a' is called the *initial point*, and $VP\ 1$ the *vanishing point*, of the line $M-N$.

Definite and Indefinite Perspective. — Perspective is spoken of as definite when it represents that which is defined within known boundaries.

The indefinite perspective of a line is the perspective of the general direction of the line, while the definite perspective of the line is the perspective of some limited portion of the line.

The indefinite perspective of a line can always be drawn when we have the initial and vanishing points of the line, or when we have the vanishing point, and the perspective of some other point in the line.

130. Initial Point. — The initial point of a line is the point in which the line intersects V .

131. Initial Trace. — The initial trace of a plane is the vertical trace of the plane.

132. Vanishing Point. — The vanishing point of a line is the point in which a line through the point of sight parallel to the given line pierces V .

The vanishing point is sometimes spoken of as the perspective of a point on the line infinitely distant back of V ; for if a visual ray should be drawn to such a point it would be practically parallel to the given line and therefore intersect V at the vanishing point.

133. Vanishing Point of a System of Lines. — If now we should attempt to find the vanishing points of a number of parallel lines, we should find them to be one and the same point; for a line drawn through the point of sight parallel to any one of the system will be parallel to all.

All the lines of a system, then, have a common vanishing point. This is an important principle, as the boundary lines of the majority of natural objects exist in systems.

134. **Vanishing Trace.** — The vanishing trace of a plane is the line in which a plane through the point of sight parallel to the given plane intersects V .

135. **The Vanishing Trace of a System of Planes** is the vanishing trace of any one of them, as but one plane through the point of sight, parallel to the system, can be drawn.

136. **Problem 47.** — *Find the initial and vanishing points of the line* $[M = -2, -3, 2; N = 1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}]$. $C = 0, 3, 2$.

137. **Problem 48.** — *Find the initial and vanishing points of the line* $[M = 2, -\frac{1}{2}, -1; N = -2, -3, 2]$. $C = 0, 4, 1$.

138. **Problem 49.** — *Find the initial and vanishing points of the line* $[M = -2, -3, 2; N = 1, -\frac{1}{2}, -1]$. $C = 0, 3, 1$.

139. **Problem 50.** — *Find the initial and vanishing points of the line* $[M = -2, -4, 3; N = -2, -1, \frac{1}{2}]$. $C = 1, 3, 1$.

140. **Problem 51.** — *Find the initial and vanishing points of the line* $[M = 2, -1, 2; N = 2, -4, 2]$. $C = -1, 3, 1$.

141. **Problem 52.** — *Find the initial and vanishing traces of the horizontal projecting plane of the line* $[M = -2, -\frac{1}{2}, 2; N = 2, -3, 2]$. $C = 0, 3, 1$.

142. **Problem 53.** — *Find the initial and vanishing traces of the plane containing the three points* $M = -2, -3, -1; N = 0, -1, 3; O = 2, -2, \frac{1}{2}$. $C = 0, 3\frac{1}{2}, \frac{1}{2}$.

143. **Problem 54.** — *Find the initial and vanishing traces of the horizontal projecting plane of the line* $[M = 1, -3, 2; N = 1, -1, 2]$. $C = -1, 3, 1$.

CHAPTER IX

PERSPECTIVE OF LINES WHICH ARE PARALLEL TO V

144. The Initial Points. — According to Section 130, lines which are parallel to V can have no initial points.

145. The Vanishing Points. — According to Section 132, lines which are parallel to V can have no vanishing points.

146. To find the Perspective, Indefinite and Definite, of Lines of this Class. — The indefinite perspective of lines of this class may be found by Section 122, and the definite perspective may be found by finding the perspectives of those points which limit definite portions of the lines.

The perspective of a system of such lines will be a set of parallel lines with an imaginary vanishing point at an infinite distance from the center of the picture.

Having, therefore, the perspective of one line of such a system, the perspective of any other line of the same system can be drawn when we know the perspective of one point in the line.

147. Problem 55. — *Find the perspective of a definite portion of the line $[M = -3, -2, 1; N = 3, -2, 3]$, when $C = 0, 4, 1\frac{1}{2}$.*

148. Problem 56. — *Find the perspective of a definite portion of the line $[M = 2, -3, 1; N = 2, -3, 4]$, when $C = 0, 4, 1\frac{1}{2}$.*

149. Problem 57. — *Find the indefinite perspective of the lines $[M = -2, -3, \frac{1}{2}; N = 2, -3, 2]$, $[O = 2, -1, \frac{1}{2}; P = 2, -1, 2]$ and $[Q = -2, -3, 1\frac{1}{2}; R = 2, -3, 3]$, when $C = 0, 4, 1$.*

CHAPTER X

PERSPECTIVE OF LINES WHICH ARE PERPENDICULAR TO V

150. **The Initial Points.** — The initial points of these lines may be found by Section 130.

151. **The Vanishing Point.** — The vanishing point of these lines will be found by Section 132 to be at c' , the center of the picture.

152. **Points of Distance.** — See Plate VI, Fig. 40. Through C draw the line $C-PD1$, parallel to H , and making an angle of 45 degrees with V . The point $PD1$, in which this line pierces V , is called a *point of distance*.

153. **Case A.** — To find the perspective, indefinite and definite, of a line of this class when the initial point is given. See Plate VI, Fig. 40.

Let a' be the initial point of the given line. Then the line $a'-c'$ will be the indefinite perspective.

Let it be required to find the definite perspective of a certain portion of the line, say three units, measured from the point a' . Draw through a' the line $a'-e'$ parallel to $H-L$. Lay off upon this line from a' three units to b' , and connect b' with $PD1$. The latter line cuts the indefinite perspective at d'' , and $a'-d''$ will be the definite perspective sought.

The line $a'-e'$ is called a *line of measures*, and the line $b'-PD1$ is called a *measuring line*.

The above proposition may be proved as follows: From the method by which the point $PD1$ was found it is evident that it is the vanishing point of that system of lines parallel to H and

inclining toward the right at an angle of 45 degrees with V . The line $a'-b'$ is in V , parallel to $G-L$, and is the perspective of itself. The angle $PD1-b'-a'$, then, is the perspective of an angle of 45 degrees; the angle $c'-a'-b'$, the perspective of an angle of 90 degrees; and the angle $a'-d'-b'$ the perspective of an angle of 45 degrees, since the figure $a'-b'-d'$ is a triangle.

The figure $a'-b'-d'$ must, then, be the perspective of an isosceles triangle in which the equal sides are $A-B$ and $A-D$. Therefore $a'-d'$ is the perspective of a distance equal to the actual distance $a'-b'$.

Distances can be laid off on the line of measures on either side of the initial point, giving rise to two measuring lines; but that particular lay-out which results in a good intersection with the indefinite perspective should be selected.

It is evident from Fig. 40 that the point d' is the perspective of a point at a known distance (viz., a' from Y) from P , three units back of V , and at a known distance (viz., a' from $G-L$) from H .

154. Rule for Case A.—To find the definite perspective of a line which is perpendicular to V : Connect the initial point with c' . Through the initial point draw a line parallel to $H-L$, and lay off upon this line from the initial point a distance equal to that to be put into perspective. Connect the point thus found with the point of distance.

155. Case B.—To find the perspective, indefinite and definite, of a line of this class when its vanishing point and the perspective of one of its points are known: A line through the given perspective of the point and c' will be the indefinite perspective of the line.

In finding the definite perspective, first determine, by the conditions of the problem, the initial trace of some plane containing the line. The intersection of this trace by the indefinite

perspective of the line will be the initial point of the line. As we now have the initial point of the line, we may proceed as in Case A, remembering to take into account the distance from the initial point to the point given in perspective. This and other methods will be illustrated in the next chapter.

156. Problem 58.—*Find the perspective of the point $M = -2, -4, 1$, when $C = 1, 4, 1\frac{1}{2}$.*

NOTE.—In this and subsequent problems find the perspective by use of the vanishing point.

157. Problem 59.—*Find the perspective of the point $M = 2, -3, 4$, when $C = -1, 4, 1\frac{1}{2}$.*

158. Problem 60.—*Find the perspective of the point $M = 1, -2, \frac{3}{4}$, when $C = -\frac{3}{4}, 2\frac{1}{4}, 1$.*

If we proceed to solve this problem by the ordinary method, we can see by Plate VII, Fig. 41, that the required point m^p is inaccurately located, since the measuring line runs so nearly parallel to the indefinite perspective of the line.

This difficulty may be avoided thus: Through the initial point a' draw a vertical line and on it select a new initial point anywhere, as f' . Through f' draw $f'-g'$ parallel to $H-L$ until it intersects the vertical through b' at g' . The line $f'-c'$ is the perspective of a line parallel to and in the same vertical plane with the line of which $a'-c'$ is the perspective. The line $g'-PD1$ is the perspective of a line parallel to and in the same vertical plane with the line of which $b'-PD1$ is the perspective. Therefore k^p is the perspective of a point in the same vertical line as m^p , and a vertical line through k^p should cut the line $a'-c'$ in the required perspective.

This principle will be somewhat extended and advantageously employed in subsequent problems.

159. Problem 61.—*Find the perspective of an inch and a quarter cube one of whose faces coincides with H and another with V ,*

and whose center is $\frac{1}{2}$ to the right of P . $C = -2, 3\frac{1}{2}, 1\frac{3}{4}$. See Plate VII, Fig. 42.

The square $a'-b'-d'-e'$ is both the vertical projection of the cube and the perspective of its front face. The four lines $a'-c'$, $b'-c'$, $d'-c'$, and $e'-c'$ are the indefinite perspectives of the four edges which are perpendicular to V . $G-L$ is a line of measures for the two lines $e'-c'$ and $d'-c'$; while $b'-a'$ produced is a line of measures for the two lines $a'-c'$ and $b'-c'$.

Lay off $e'-m'$ and $a'-n'$ each equal to $1\frac{1}{2}$. Connect m' with $PD\ 1$, locating the point f^p . A line from e' to $PD\ 1$ locates l^p , one from n' to $PD\ 1$ locates g^p , and one from a' to $PD\ 1$ locates k^p .

After finding the point f^p the remainder of the drawing can be easily made without the use of measuring lines. The lines $A-B$, $E-D$, $F-L$, and $G-K$ constitute a system parallel to V , and their perspectives must be parallel. The perspectives of $A-E$, $B-D$, $K-L$, and $G-F$ must also be parallel.

Therefore through f^p draw f^p-l^p parallel to $e'-d'$ to meet $d'-c'$ in l^p . Through f^p draw f^p-g^p parallel to $e'-a'$ to meet $a'-c'$ in g^p . Through g^p draw g^p-k^p parallel to $a'-b'$ to meet $b'-c'$ in k^p . Finally, through k^p draw k^p-l^p parallel to $b'-d'$ to pass through l^p .

160. Problem 62.—Find the perspective of a rectangular prism resting on H , $\frac{1}{2}$ to the left of P , $\frac{2}{3}$ back of V , and having its shortest dimensions parallel to V . Altitude of the prism, $1\frac{3}{4}$; base, $1\frac{1}{2} \times \frac{3}{4}$. $C = 1\frac{1}{4}, 3\frac{1}{2}, 1$. See Plate VII, Fig. 43.

The rectangle $a'-b'-(d')-(e')$, is the vertical projection of the front face of the prism, or it is the position which this face would occupy were the prism brought back, perpendicularly to V , until it coincided with V .

The line $G-L$ serves as a line of measures, and the points d^p , l^p , etc., are located as above.

The figure also shows an application of the principle of Section 158. An auxiliary horizontal plane whose ground line

is $G'-L'$ is assumed below H . This plane may be assumed above or below H and at any distance from it.

The figure $d_{11}^p-l_{11}^p-f_{11}^p-e_{11}^p$ is the perspective of the plan or projection of the prism upon this auxiliary plane, and can be found when we know the position of the prism with reference to P and V .

The lines $d_{11}^p-b^p$, $e_{11}^p-a^p$, $f_{11}^p-q^p$, and $l_{11}^p-k^p$ are vertical lines through d_{11}^p , e_{11}^p , f_{11}^p , and l_{11}^p respectively, and must coincide with the vertical edges of the prism in perspective.

The lines $(d')_{11}-c'$, $(e')_{11}-c'$, $a'-c'$, and $b'-c'$ must coincide with four of the horizontal edges of the prism in perspective and therefore cut the vertical lines previously drawn in the required vertexes.

The student is strongly advised to employ this principle in subsequent problems.

The perspective of the plan may be made upon an auxiliary piece of paper pinned over the regular plate, thus relieving and protecting the surface. This auxiliary piece may be placed below or even above the ground line, and of course removed after the plate has been completed.

161. Problem 63. — *Find the perspective of an inch and a half cube, $\frac{1}{2}$ to the left of P , $\frac{3}{4}$ back of V , $\frac{3}{4}$ above H , and having its faces parallel and perpendicular to H and V . $C = 2, 3, 1\frac{1}{2}$.*

CHAPTER XI

PERSPECTIVE OF LINES WHICH ARE PARALLEL TO *H* BUT OBLIQUE TO *V*

162. The Vanishing Points. — See Plate VII, Fig. 44. Having given the direction of the line, draw through *C* a line, *C-VP* 1, parallel to this line. The point *VP* 1 in which the last line intersects *V* is the vanishing point of the given line.

163. The Measuring Points. — See Plate VII, Fig. 44. Revolve *C* about the line *VP* 1-*vp* 1, as an axis until *C* falls in *V*. The point *MP* 1 in which *C* falls is the measuring point of the given line. It will be observed that the distance of the measuring point from the vanishing point is the same as the distance of the observer from the vanishing point.

164. Case A. — To find the perspective, indefinite and definite, of a line of this class when the initial point and the inclination to *V* are given. See Plate VII, Fig. 44. Let *a'* be the initial point. Then the line *a'-VP* 1 will be the indefinite perspective of the line.

Let it be required to find the definite perspective of a certain portion of this line, say three units, measured from *a'*. Draw through *a'* the line *a'-e'* parallel to *H-L*. Lay off upon this line from *a'* three units to *b'*, and connect *b'* with *MP* 1. The latter line cuts the indefinite perspective at *d^p*, and *a'-d^p* will be the definite perspective sought.

Proof. — Connect *c*, and *mp* 1, completing the isosceles triangle whose equal sides are *vp* 1,-*mp* 1, and *vp* 1,-*c*, respectively. Call the angle which the given line makes with *V*, α ; the

angle $vp\ 1, -c, -mp\ 1, \beta$; and the angle which $c, -mp\ 1$, makes with $G-L$, γ .

Lines of this class are parallel to H , therefore their vanishing points are in $H-L$. By construction, $VP\ 1$ is the vanishing point of all lines of this class extending backward toward the right and making the given angle α with V .

The point $MP\ 1$ is for the same reason the vanishing point of all lines of this class extending backward toward the left and making the angle γ with V . Then the angle $b'-a'-d^p$ is the perspective of the angle α . Angle $a'-b'-d^p$ is the perspective of the angle γ . Angle $a'-d^p-b'$, which is the remaining angle of the triangle, must be the perspective of the supplement of the angle $\alpha + \gamma$, which is β . But by the diagram, the angle β equals the angle γ . Therefore the triangle $a'-d^p-b'$ is the perspective of an isosceles triangle in which the equal sides are $A-D$ and $A-B$. The line $a'-d^p$ is, then, the perspective of three units of the given line measured from a' .

165. Rule for Case A.—To find the definite perspective of a line which is parallel to H but oblique to V : Connect the initial point with the vanishing point. Through the initial point draw a line parallel to $H-L$, and lay off upon this line from the initial point a distance equal to that to be put into perspective. Connect the point thus found with the measuring point.

166. Problem 64.—Draw the perspective of a square prism, $1\frac{1}{2} \times 1\frac{1}{2} \times 2\frac{1}{2}$. One edge of the prism coincides with V and is directly in front of C . The prism rests on H , and its right-hand face makes an angle of 30 degrees with V . $C = 0, 2\frac{1}{2}, 1$. See Plate VII, Fig. 45.

The edges of the prism which are not parallel to V are parallel to H , and the problem comes under this chapter.

Knowing the angles which the faces, and consequently the edges of the prism, make with V , the vanishing and measuring points are found as in Sections 162 and 163.

The line $A-B$ is the edge coincident with V .

The point a' is the initial point of the two adjacent edges $A-F$ and $A-M$. The points $VP\ 1$ and $VP\ 2$ respectively are the vanishing points of these two edges. The line $a'-f^p$ is the definite perspective of $1\frac{1}{2}$ units, and is found according to Section 164.

The definite perspective $a'-m^p$ is found in the same way, and the remainder of the figure is easily completed. The shade lines of the prism are $A-F$, $F-G$, $G-O$, $O-K$, $K-M$, and $M-A$, and are represented in perspective by heavy lines.

167. Case B. — To find the perspective, indefinite and definite, of a line of this class when the perspective of a point in the line and the direction of the line are given.

First Method. — See Plate VII, Fig. 46. Let a^p be the given perspective of the point and let α be the given angle which the line makes with V . Then $a^p-VP\ 1$ is the indefinite perspective of the line.

Let it be required to find the perspective of a definite portion of the line measured from the point of which a^p is the perspective. By the conditions of the problem determine the initial trace of some plane containing the line. Suppose, for example, that the distance of A above H is known. Then a line, $d'-f'$, parallel to $G-L$ and at a distance above it equal to the distance of A above H , will be the initial trace of a horizontal plane containing the line in question.

This line $d'-f'$ must contain the initial points of all lines in the horizontal plane through A .

The line $H-L$ must contain the vanishing points of all such lines.

Through a^p and $MP\ 1$ draw a line and produce it until it intersects V in e' . Lay off upon the line $d'-f'$, from e' , the distance $e'-f'$, equal to that distance whose perspective is sought. Connect f' with $MP\ 1$. The latter line cuts the indefinite perspective at b^p , and a^p-b^p is the desired perspective.

Proof. — Produce the line $VP\ 1-a^p$ until it intersects V in d' . The line $d'-a^p$ is by rule the perspective of the distance $d'-e'$; the line $d'-b^p$ is for the same reason the perspective of the distance $d'-f'$. Therefore a^p-b^p , which is $d'-b^p$ minus $d'-a^p$, must be the perspective of the distance $e'-f'$, which is $d'-f'$ minus $d'-e'$.

Or, the triangle $d'-b^p-f'$ is, by Case A, the perspective of an isosceles triangle. The line $f'-b^p$ is the perspective of the base. The line $e'-a^p$ is the perspective of a line parallel to the base and therefore cutting off equal distances on the equal sides.

Second Method. — See Plate VII, Fig. 47. As in the first method, a^p is the given perspective of the point and α is the given angle. Then $a^p-VP\ 1$ is the indefinite perspective of the line.

In finding the definite perspective, let us suppose again that the distance of the point A above H is known. Then the line $d'-f'$, drawn under the same conditions as in the first method, must contain the initial points of all lines in the horizontal plane through A .

Through a^p draw a^p-b^p parallel to $H-L$. This line is the perspective of a horizontal line through A parallel to V .

Draw the line $c'-a^p$ and produce it to intersect V at a' .

Lay off upon $d'-f'$ from a' the distance $a'-b'$ equal to the distance whose perspective is sought. Connect b' and c' , cutting the line a^p-b^p at b^p . Draw $b^p-MP\ 1$, cutting $a^p-VP\ 1$ at d^p . The line a^p-d^p is the definite perspective sought.

Proof. — According to Case A, the triangle $a^p-b^p-d^p$ is the perspective of an isosceles triangle of which $A-B$ and $A-D$ are the equal sides. But a^p-b^p is the perspective of the distance $a'-b'$, parallels included between parallels. Therefore a^p-d^p is the perspective of the distance $a'-b'$.

168. Problem 65. — *Draw the perspective of a rectangular pillar $1\frac{1}{2} \times 1\frac{1}{4} \times 2\frac{1}{2}$, surmounted by a rectangular abacus $1\frac{1}{4} \times 2\frac{1}{2} \times \frac{3}{8}$.*

The pillar rests on H , with its front edge 1 unit to the left of the observer and $\frac{1}{2}$ unit back of V . The two faces adjacent to the front edge of the pillar make angles of 30 degrees and 60 degrees respectively with V . $C = 0, 2\frac{1}{2}, 1\frac{1}{2}$. See Plate VIII, Fig. 48.

Find the point of distance, vanishing points, and measuring points as before. If the edge of the pillar were in V , one unit to the left of the observer, it would fall on a'_1-h' . Select any point in a'_1-h' produced, as a_{11}' , and through it draw $G'-L'$ as a new ground line.

The point b_{11}^p is the perspective of a point one unit to the left of C and one half a unit back of V (see Section 154), and therefore directly beneath the front edge of the pillar in its true position. Through b_{11}^p raise a vertical to intersect a'_1-c' in b^p . The point b^p is the perspective of the lower front vertex of the pillar (see Section 158).

Upon the point b_{11}^p complete the perspective of the plan of the pillar whose dimensions are shown in Plate VIII, Fig. 48 *a*. The line b_{11}^p-VP1 is the indefinite perspective of one side of this plan, and b_{11}^p-VP2 is the indefinite perspective of its adjacent side. Draw $MP1-b_{11}^p$ and produce it to intersect $G'-L'$ in e_{11}' . Lay off upon $G'-L'$ from e_{11}' the distance $e_{11}'-f_{11}'$ equal to $1\frac{1}{2}$ and connect f_{11}' with $MP1$. The line $b_{11}^p-g_{11}^p$ is the definite perspective of one side of the plan.

The lines b^p-VP1 and b^p-VP2 are the indefinite perspectives of two sides of the base of the pillar in true position. Therefore the verticals through g_{11}^p and r_{11}^p determine the definite perspectives of these two sides.

Lay off upon a'_1-h' from a'_1 to h' the altitude of the prism. Connect h' with c' . The point s^p is the perspective of the front vertex of the upper base of the pillar. The perspective of the pillar is now easily completed.

To draw the perspective of the abacus: A comparison of the horizontal dimensions of the abacus with the horizontal

dimensions of the pillar shows that there will be an overlap of one fourth all around.

Lay off upon $G'-L'$ from e_{11}' the distance $e_{11}'-k_{11}'$ equal to one fourth. Connect k_{11}' with $MP\ 1$, crossing $VP\ 1-b_{11}^p$ produced at l_{11}^p . $VP\ 2-l_{11}^p-v_{11}^p$ is the indefinite perspective of one side of the plan of the abacus. In the same way q_{11}^p , u_{11}^p , and n_{11}^p are found and the perspective of the plan completed.

Produce $VP\ 2-v_{11}^p$ to t_{11}' . Raise the vertical $t_{11}'-y'$ to cross $G-L$ at t' . Lay off $t'-x'$ equal to the altitude of the pillar, and lay off $x'-y'$ equal to the altitude of the abacus. Connect x' with $VP\ 2$ and connect y' with $VP\ 2$.

A vertical through v_{11}^p cuts these last two lines at v^p and z^p respectively. The line v^p-z^p will be the perspective of the front edge of the abacus. The remainder of the drawing is now easily completed by aid of the perspective of the plan.

NOTE. — All measurements must be made upon the picture plane (where each distance is its own perspective), and then by the rules of perspective be transferred to their respective positions back of the picture plane.

169. Problem 66. — *Draw the perspective of a block of four steps whose outside dimensions are 8 feet long by 4 feet wide by 4 feet high. The block has at each end a wall 1 foot thick. The rise and tread of each step is 1 foot. The front edge of the block is 1 foot to the left of C and 1 foot back of V. The front face of the block extends backward toward the left and makes an angle of 30 degrees with V. $C = 0, 9', 5'$. See Plate IX, Fig. 49.*

The point d^p is the perspective of the lower front vertex of the block, and d_{11}^p is the perspective of the same point projected upon a horizontal plane 6 feet below H . The figure $d_{11}^p-f_{11}^p-l_{11}^p-h_{11}^p$ is the perspective of the plan of the block made upon a horizontal plane 6 feet below H .

Upon $b_{11}'-b'-k'$, which is in V , lay off from b' to k' 4 feet, the altitude of the block.

Draw $k'-PD1$, cutting a vertical through d^p at l^p . The point l^p is the perspective of the upper front vertex of the block. The indefinite perspective of the edges may now be drawn and the definite perspectives determined by verticals from the perspective of the plan.

The perspective of the outline of the steps may be found as follows: Take, for example, the intersection of the rise of the second step with the tread of the second. The perspective of the plan of this line is $m_{11}^p-n_{11}^p$. A vertical through m_{11}^p must contain the perspective of the intersection of the line in question with the right-hand face of the block. Lay off upon b'_-k' , from b'_- to o' , 2 feet, which is the height of $M-N$ above H . Connect o' with $PD1$, cutting d^p-l^p at p^p .

Connect p^p with $VP2$, cutting the vertical through m_{11}^p at m^p . The line m^p-VP1 is the perspective of the line in question. In this way the remaining lines of the steps may be found.

CHAPTER XII

PERSPECTIVE OF LINES WHICH ARE OBLIQUE TO BOTH *H* AND *V*

170. The Vanishing Points. — See Plate VIII, Fig. 50. Having given the direction of the line, draw through *C* the line *C-VP* 3 parallel to the given line. The point *VP* 3, in which the last line intersects *V*, is the vanishing point of the given line.

171. The Measuring Points. — See Plate VIII, Fig. 50. Through *VP* 3 draw any line, as *VP* 3-*MP* 3. Lay off upon this line from *VP* 3 the distance of *VP* 3 from *C*. This gives the point *MP* 3 as the measuring point of the given line.

From the diagram it may be seen that the distance of *VP* 3 from *C* may be found by revolving *C* about *VP* 3-*MP* 3 as an axis until it falls into *V*.

172. Case A. — To find the perspective, indefinite and definite, of a line of this class when the initial point and the direction of the line are known. See Plate VIII, Fig. 50. Let *a'* be the initial point. Then *a'-VP* 3 will be the indefinite perspective of the line.

Let it be required to find the definite perspective of a certain portion of this line measured from *a'*. Draw through *a'* the line *a'-b'* parallel to *VP* 3-*MP* 3. Lay off upon this line from *a'* to *b'* a distance equal to that distance whose definite perspective is sought. Connect *b'* with *MP* 3, cutting the indefinite perspective at *d''*. The line *a'-d''* is the definite perspective sought.

Proof. — Connect *c''* and *MP* 3, completing the isosceles triangle whose equal sides are *VP* 3-*MP* 3 and *VP* 3-*c''*.

The line $a'-VP\ 3$ is the perspective of a line parallel to $C-VP\ 3$. The line $b'-MP\ 3$ is the perspective of a line parallel to $C-MP\ 3$. The line $b'-a'$ is the perspective of a line parallel to $VP\ 3-MP\ 3$. But the triangle $C-VP\ 3-MP\ 3$ is isosceles, as may be seen from its revolved position $c'-VP\ 3-MP\ 3$; therefore the triangle $b'-a'-d^p$ is the perspective of an isosceles triangle in which the equal sides are $a'-b'$ and $a-D$.

173. **Case B.** — To find the perspective, indefinite and definite, of a line of this class when the perspective of a point in the line and the direction of the line are given. See Plate IX, Fig. 51.

Let d^p be the given perspective of the point. Let $VP\ 3$, found as in Section 170, be the vanishing point of the line. Then $d^p-VP\ 3$ will be the indefinite perspective of the line.

Let it be required to find the perspective of a definite portion of the line measured from the point of which d^p is the perspective. By the conditions of the problem, determine the initial point a' . Through a' draw $a'-e'$ parallel to $VP\ 3-MP\ 3$. Draw $MP\ 3-d^p$ and produce it to meet V in b' .

Lay off on $a'-e'$ from b' the distance $b'-e'$, equal to the distance whose perspective is sought. Connect e' and $MP\ 3$, cutting $a'-VP\ 3$ at f^p . The line d^p-f^p will be the perspective sought.

Proof. — By Case A, the triangle $a'-e'-f^p$ is the perspective of an isosceles triangle. The line $e'-f^p$ is the perspective of the base. The line $b'-d^p$ is the perspective of a line parallel to the base and therefore cutting off equal distances on the equal sides.

174. **Problem 67.** — *Find the perspective of a box having an open lid which inclines backward from the observer and makes an angle of 60 degrees with H. The horizontal dimensions of the box are $3\frac{1}{4}$ feet by $1\frac{1}{2}$ feet. The altitude of the box is $1\frac{1}{4}$ feet. The outside depth of the lid is $\frac{3}{4}$ feet. The stock of which the box and cover are made is 2 inches. The front edge of the box is 1 foot to the right of the observer and 6 inches back of V. The long*

dimensions of the box extend backward to the right at an angle of 30 degrees with V . $C = 0, 4\frac{1}{4}, 2'$. See Plate X, Fig. 52.

The points $VP\ 1$, $VP\ 2$, $MP\ 1$, $MP\ 2$, and $PD\ 1$ are found and the perspective of the box determined by methods previously explained.

The horizontal dimensions of the lid, if extended, will vanish at $VP\ 1$. The oblique dimensions of the lid, existing in planes parallel to the face $A-B$ of the box, must vanish in the vanishing trace of this face, which is $VP\ 3-VP\ 4$.

As the lid is inclined 60 degrees to H , the oblique dimensions of the lid must make angles of 60 degrees and 30 degrees respectively with H .

If the visual plane whose vertical trace is $VP\ 3-VP\ 4$ be revolved about this trace as an axis into V , C will fall at $MP\ 2$. The lines $MP\ 2-VP\ 3$ and $MP\ 2-VP\ 4$, which are drawn at angles of 60 degrees and 30 degrees respectively with $H-L$, will be the revolved positions of two lines drawn through C parallel respectively to the two sets of dimensions in question.

Therefore $VP\ 3$ and $VP\ 4$, the points in which these lines pierce V , will be the required vanishing points.

The points $MP\ 3$ and $MP\ 4$, determined as in Section 171, will be the required measuring points.

Connect b'' with $VP\ 3$, also b'' with $VP\ 4$. Now, since the line $f'-d'$ is the initial trace of the face $A-B$ of the box, if we produce the line $MP\ 3-b''$ it must intersect V at b' .

Lay off upon this initial trace from b' the distance $b'-d'$, equal to the width of the box, which is also the width of the lid, and connect d' with $MP\ 3$. In this way the point d'' is located.

Produce the line $MP\ 4-b''$ to intersect V at e' . Lay off $e'-f'$ equal to the depth of the lid and connect f' with $MP\ 4$. In this way the point f'' is located.

The remaining lines are now easily drawn.

175. Problem 68.—Draw the perspective of a four-inch cube. One vertex of the cube touches V , 3 inches to the right of C , and 4 inches above H . The three edges of the cube which intersect at this point have the following positions: the first extends upward and directly backward; the second extends downward, backward, and to the right; and the third extends downward, backward, and to the left.

CHAPTER XIII

PERSPECTIVE OF CURVES

176. Theory. — The perspective of a curve is the intersection with the picture plane of a conical visual surface made up of an infinite number of visual rays through the points of the curve.

In some of the simple curves the perspectives may be found by an application of the principles of conic sections, but, as a general rule, the problem can be more easily solved by finding the perspectives of a number of important points and tangents of the curve.

177. Problem 69. — *Find the perspective of an inch and a half circle. The surface of the circle is vertical, intersects V $1\frac{1}{2}$ inches to the right of P , and is inclined 60 degrees to V . $C = 0, 3\frac{1}{2}, 1$.*

In Plate VIII, Fig. 53, $A-B-D-E$ is the plan of the circle inscribed in a square. Q, G, R, S , etc., are any points assumed in the circumference.

The figure $a'-b'-d''-e''$ is the perspective of the circumscribing square in its required position. The points $q'', g'', r'', s'',$ etc., are the perspectives of the points Q, G, R, S , etc., whose real positions with reference to the sides of the circumscribing square may be determined from the plan.

By the aid of the perspectives of the four tangent sides and of the perspectives of a sufficient number of points in the circumference, the required curve may be traced.

178. Problem 70. — *Find the perspective of an irregular plane curve whose surface is vertical and inclined to V .*

In Plate X, Fig. 54, let $T-V-D-L-I$ be the irregular curve. Refer the important points, by coördinates, to two rectangular axes, X and Y , which, as in this case, may often be drawn tangent to the curve.

Place the axes and their coördinate measuring lines in their required position in perspective and trace the curve through the points thus located.

The line $o'-y'$ is the perspective of the axis of Y , and $o'-x'$ is the perspective of the axis of X .

The lines $k'-VP\ 1$, $m'-VP\ 1$, etc., are the perspectives of the abscissas, while $b'-j'$, $e'-t'$, etc., are the perspectives of the ordinates.

179. Use of Coördinate Axes. — It is evident from the last problem that any irregular plane curve or polygon may be referred to coördinate axes and that the perspective of the curve may be found by referring the curve to the perspective of these axes.

In the same way any irregular curve of double curvature may be referred to three coördinate axes, and the perspective of the curve may be found by referring the curve to the perspective of these three axes.

CHAPTER XIV

PERSPECTIVE OF SHADOWS

180. Theory. — The shadow of a point upon a surface is the intersection with that surface of a ray of light through the point; or, it is the intersection of the ray with the trace upon the surface of any plane containing the ray.

The perspective of the shadow of a given point upon a given surface is the intersection of the perspective of the ray containing the point with the perspective of the trace upon the surface of any plane containing the ray. If the projecting plane of the ray be taken as the plane containing the ray, the trace upon the surface will be the projection of the ray upon the surface.

181. The Vanishing Point of Rays. — Rays of light are parallel lines and therefore have a common vanishing point. See Plate IX, Fig. 55.

Through the point of sight draw a line parallel to a ray of light. This line pierces V at VPR and is therefore the point sought.

182. The Vanishing Point of the Projections of Rays. — The projections of rays upon any plane will be parallel lines and will have a common vanishing point, but the position of this vanishing point will depend upon the position of the plane upon which the projections are made.

183. The Vanishing Point of the Projections of Rays upon H . — The vanishing point of the projections of rays upon H , or upon any horizontal plane, will be the point where a line through C , parallel to the horizontal projection of a ray, pierces V . When

the rays have the conventional direction, as in Plate IX, Fig. 55, this point ($VPPR$) will be upon $H-L$ and as far to the right of c' as C is from c' .

184. The Perspective of the Projections of Rays upon V . — The projections of rays upon V , or upon any plane parallel to V , will have no vanishing point (see Section 145). The perspectives of such projections, however, will be parallel to the vertical projection of a ray (see Section 146).

185. The Vanishing Point of the Projections of Rays upon P . — The vanishing point of the projections of rays upon P will be upon a vertical through c' and, in case the rays have the conventional direction, as far below c' as C is from c' .

186. The Vanishing Point of the Projections of Rays upon any Plane. — The vanishing point of the projections of rays upon any plane may be found by projecting upon this plane a ray and finding the intersection with V of a line drawn through C parallel to this projection.

187. Problem 71. — *Find the perspective of the shadow upon H of the point $M = -1\frac{1}{4}, -\frac{3}{4}, 1\frac{1}{2}$, when $C = \frac{1}{4}, 1\frac{1}{4}, \frac{3}{4}$.* In Plate IX, Fig. 56, VPR and $VPPR$ are found as above. The point m^p is the perspective of M . The line m^p-VPR is the perspective of a ray through M , and the line m^p-VPPR is the perspective of the horizontal projection of the same ray. Therefore m_s^p , the intersection of these two lines, is the required point.

188. Problem 72. — *Find the perspective of the shadow upon H of the line [$M = -2, -1, \frac{1}{2}$; $N = 2, -3, 2$], when $C = 0, 3, 2$.* Find the perspectives of the shadows of the two points. The line passing through these two points in perspective will be the required line.

189. Problem 73. — *Find the perspective of the shadow upon H of a square card whose surface is parallel to H and above it.* Find the perspectives of the shadows of the four vertexes of the card and connect the points so found.

190. Problem 74.—Find the perspective of the shadow upon H of a square card whose surface is perpendicular to H and above it.

191. Problem 75.—Find the perspective of the shadow upon H of a cube which is above H and whose surfaces are parallel and perpendicular to V . Find the perspectives of the shadows of the shade lines of the cube.

192. Problem 76.—Find the perspective of the shadow upon H of an hexagonal prism which is above H and whose edges are perpendicular to H .

193. Problem 77.—Find the perspective of the shadow upon a plane parallel to H and above it of the point $M = -1\frac{1}{2}, -\frac{3}{4}, 1\frac{1}{2}$. $C = 0, 2\frac{1}{2}, 1\frac{1}{2}$.

In Plate XI, Fig. 57, $G'-L'$ is the vertical trace of the new plane. The point m^p is the perspective of the point M , and m_{11}^p is the perspective of its horizontal projection. The point m_{111}^p is the perspective of the projection of M upon the new plane.

The line m_{111}^p-VPPR is the perspective of the projection upon the new plane of a ray through M , and ms^p is the required point.

194. Problem 78.—Find the perspective of the shadow upon a plane parallel to H and $\frac{1}{2}$ inch above it of the line [$M = -3, -4, 1\frac{1}{2}$; $N = 2, -1, 1\frac{1}{2}$]. $C = 0, 4, 2$.

195. Problem 79.—Find the perspective of the shadow upon H and upon a plane below H of a square card whose surface is parallel to H and above it.

196. Problem 80.—Find the perspective of the shadow upon a plane parallel to V and $1\frac{1}{2}$ inches back of V of the point $M = -1\frac{1}{4}, -\frac{1}{2}, 1\frac{1}{2}$. $C = -\frac{1}{4}, 2, 1\frac{1}{2}$.

In Plate X, Fig. 58, $G'-L'$, is the perspective of the horizontal trace of the plane parallel to V and back of V . The point m^p is the perspective of M . The point mt^p is the perspective of the orthographic projection of M upon the new plane. The line m^p-VPR is the perspective of a ray through M , and mt^p-at^p ,

drawn through mt^p parallel to the vertical projection of a ray, is the perspective of the projection upon the new plane of the ray through M . The point ms^p , the intersection of the last two lines, is the perspective of the shadow sought.

197. Problem 81. — *Find the perspective of the shadow upon an oblique plane of a point $M = -1\frac{1}{2}, -\frac{3}{4}, 1\frac{1}{2}$. $C = -\frac{1}{2}, 2, 1$.*

In Plate XI, Fig. 59, the point m^p is the perspective of M , and m_p^p is the perspective of its horizontal projection. $T-t^p$ is the perspective of the horizontal trace of the given plane, and $t''-T-t'$ is the perspective of its vertical trace.

The line $S-m^p-VPPr$ is the perspective of the horizontal trace of the horizontal projecting plane of the ray through M , and $S-s'$ is the perspective of the vertical trace of the same plane.

The point n^p is the perspective of the intersection of the horizontal traces of the two planes, and is therefore the perspective of one point of the intersection of the planes.

The point o' is the perspective of the intersection of the vertical traces of the same planes and is therefore the perspective of another point in the intersection of the planes.

The line $o'-n^p$ is the perspective of the intersection of the two planes, and ms^p , the point where this intersection is crossed by the perspective of the ray through M , is the perspective of the shadow sought.

198. Problem 82. — *Given the perspective of a square pillar with a square pedestal; find the perspective of the shadow of the pillar upon the pedestal, and the perspective of the shadows of both upon H .*

199. Problem 83. — *Given a square pillar casting a shadow upon a pyramidal capped slab with pedestal; required the perspective of the shadows. See Plate XI, Fig. 60.*

The perspective of the given magnitudes, the vanishing point of rays, and the vanishing point of the projections of rays may be found as previously explained.

The needed shade lines of the pillar are $A-B$, $B-D$, $D-E$, and $E-F$. The shadows of $A-B$ and $E-F$ upon the several surfaces may be determined by finding the intersections with these surfaces of planes of rays through $A-B$ and $E-F$.

The perspective of the horizontal trace of the plane of rays through $A-B$ is a^p-VPPr .

The line g^p-k^p is the perspective of the intersection of the same plane with the front face of the pedestal.

The line l^p-m^p is the perspective of the intersection of the same plane with the front face of the slab.

The line m^p-u^p-VPPr is the perspective of the intersection of the same plane with the upper horizontal surface of the slab.

To find the perspective of the intersection of the plane of rays through $A-B$ with the left front face of the pyramid: $G'-L'$ is the initial trace of the plane of the upper horizontal surface of the slab. Regarding this plane for the moment as H , the line $S-s^p$, which is the production of the line z^p-y^p , will be the perspective of the horizontal trace of the plane of the left front face of the pyramid.

The line $U-Z$ vanishes at $PD1$. Therefore x^p-PD1 will be the perspective of a line through the apex X , parallel to $U-Z$, and thus be a line of the plane of that face of the pyramid in question. This last line intersects V in the point s'' in the line $G''-L''$, which is the initial trace of a horizontal plane through X . The line $s''-S-s'$, then, is the vertical trace of the plane of the left front face of the pyramid.

$T-m^p-u^p-t^p-VPPr$ will be the perspective of the horizontal trace of the plane of rays through $A-B$. $T-t'$, drawn perpendicular to $G'-L'$, will be the vertical trace of the same plane.

The two horizontal traces now determined intersect in perspective at u^p , and the two vertical traces intersect in perspective at the point w' , not shown in the drawing. Therefore the line $w'-u^p$ produced is the perspective of the intersection sought.

The line b^p-VPR is the perspective of a ray through B , and its intersection with the line $w'-u^p$ produced, which is bs^p , is the perspective of the shadow of B upon this same face of the pyramid.

In the same way the perspective of the shadows of D and E upon this face may be found.

The perspective of the shadow of the slab upon the pedestal and the perspective of the shadow of both upon H may be found by an application of principles already explained.

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